

Indirect Measurement with Shadows (Similar Triangles)

Feature	Detail
Topic	Indirect Measurement using Similar Triangles and Shadows
Time	60–90 minutes (can be split across two days)
Grade Level	Middle School (Grades 7–9)
Prerequisites	Understanding of Ratios, Proportions, and Basic Triangle properties.
Learning Objective	Students will be able to apply the properties of similar triangles to calculate the height of an object that cannot be measured directly.

Materials

- **For the Class:** Projector, whiteboard/smartboard.
 - **For Each Group (3-4 students):**
 - Measuring Tape (at least 25-50 feet)
 - Ruler or Meter Stick
 - Pencil and Clipboard/Paper
 - Calculator (optional, but encouraged)
 - *Alternative:* A known reference object (e.g., a yardstick or a person of known height).
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Procedure

Part 1: Introduction and Theory (20 Minutes)

1. **Engage (The Un-Measurable Problem):**
 - Ask students: “How could you measure the height of the school flagpole/tree/building without climbing it?”
 - Allow a few minutes for brainstorming (they might suggest dropping a string, using drones, etc.). Guide the discussion toward methods using ratios and shadows.
 - Introduce **Thales of Miletus** (an ancient Greek mathematician) and his use of shadows to calculate the height of the Egyptian pyramids.
2. **Theory: Similar Triangles and Proportions:**
 - Draw a large diagram on the board showing a person standing next to a tall object (like a tree) on a sunny day.
 - *The key insight:* The sun’s rays are essentially parallel, so the angle at which the ray hits the top of the object and the top of the person’s head is the same.
 - Draw the right triangles formed by:
 - **Triangle 1:** (Person’s Height) + (Person’s Shadow)
 - **Triangle 2:** (Object’s Height) + (Object’s Shadow)
 - **Justification of Similarity:**
 - Angle 1: Both objects are assumed to be perpendicular to the ground (**90° right**

- angle).
- Angle 2: The angle of the sun's rays is the same for both objects (**corresponding angles are equal**).
- Conclusion: Since two angles are equal (AA Similarity Postulate), the triangles are **similar**.
- **Set up the Proportion:** Since the triangles are similar, their corresponding sides are proportional. Person's Shadow (s)Person's Height (h)
=Object's Shadow (S)Object's Height (H)or Object's Height (H)Person's Height (h)
=Object's Shadow (S)Person's Shadow (s)

Part 2: Fieldwork and Data Collection (30 Minutes)

1. Preparation and Group Assignment:

- Form small groups (3-4 students).
- Review the materials and ensure students know how to use the measuring tape accurately.
- Choose a tall, stationary object on the school grounds (flagpole, light post, a tree). *Make sure the shadow is visible on flat ground.*

2. Data Collection Steps:

- **Step 1: Choose Your Reference Person (The "Human Meter Stick"):** One student is the "reference object." Measure their height and record it. (Encourage them to stand up straight!)
- **Step 2: Measure the Reference Shadow:** Have the reference person stand perfectly still. Measure the length of their shadow *at that exact time*. **Crucially, all measurements must be taken at the same time!**
- **Step 3: Measure the Object's Shadow:** Measure the total length of the tall object's shadow.
- **Data Recording:** Have students record their data clearly in a table:

Measurement	Value	Unit
h (Reference Person's Height)		
s (Reference Person's Shadow)		
S (Object's Total Shadow)		
H (Object's Height)	Unknown	

Part 3: Calculation and Conclusion (25 Minutes)

1. Return to the Classroom:

Bring students back inside to complete the calculations.

2. Calculation:

- Have each group plug their measured values into the proportion: $sh=SH$
- Solve for the unknown height, **H**: $H=sh \cdot S$

3. Analysis and Reflection:

- Have each group post their calculated height (**H**) on the board.
- **Discussion Questions:**
 - "How close were the results across the different groups?" (This introduces the concept of **measurement error**).
 - "What factors might have caused differences in the answers?" (Potential answers: imperfect measuring, not standing perfectly straight, ground not perfectly level,

- not measuring at the *exact* same moment).
- “Do your answers seem reasonable? Why or why not? (i.e., The flagpole is 12 feet, not 120 feet.)”
 - “If we did this again at a different time of day, what would change? (The shadows s and S would change, but the ratio $\mathbf{h/s}$ should be equal to $\mathbf{H/S}$ at that new time).”
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Extension Activity (Optional)

- **Mirror Method:** Introduce an alternative method that also uses similar triangles but does not depend on shadows. Groups use a small, flat mirror placed on the ground between the reference person and the tall object. The person backs away until they can see the very top of the object reflected in the center of the mirror. The proportion involves:

$$\frac{\text{Distance from Person to Mirror}}{\text{Person's Eye Height}} = \frac{\text{Distance from Mirror to Object}}{\text{Object's Height}}$$
- **Trigonometry Connection (High School):** For more advanced classes, after finding the height, have them use the tangent function to calculate the **angle of elevation** of the sun at the time of the measurement. $\tan(\theta) = \frac{\text{Person's Height}}{\text{Person's Shadow}} \implies \theta = \tan^{-1}\left(\frac{h}{s}\right)$ This reinforces why the angle is the same for both triangles.